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FINAL SCIENTIFIC REPORT, GRANT NUMBER AFOSR-78-3599.(U)
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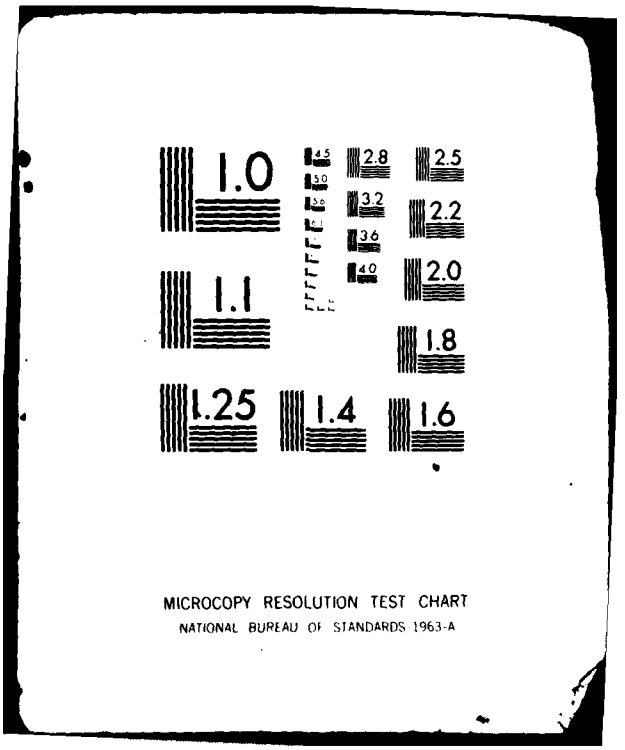
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The major goal of the grant was to see if an algorithm for finding a polynomial of best approximation with integral coefficients could be developed. This is what is actually needed in digital filter design. This report summarizes the results produced under the grant.

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AFOSR-TR- 82-0085

FINAL SCIENTIFIC REPORT
Grant No. AFOSR 78-3599 , 6/1/78 to 5/31/81

by Le Baron O. Ferguson
Principal Investigator

Two papers were written during the period covered by the grant. In addition, work was completed on a book which is now in print. These three items will now be covered.

The first paper is "Uniform and L_p approximation for generalized integral polynomials" which was published in the Pacific Journal of Mathematics, Vol. 84, No. 1, 1979, pp. 53-62. Suppose that X is a compact Hausdorff space and let $C(X,\underset{\sim}{R})$ denote the space of real valued continuous functions with norm defined by $\|f\| = \sup_{x\in X}|f(x)|$. A subset F of $C(X,\underset{\sim}{R})$ is said to be point separating if for any two distinct points x and y in X there is an f in F such that $f(x)\neq f(y)$. Let μ be a finite regular positive Borel measure on X and let $L_p(\mu)$ $(1\leq p<\infty)$ denote the space of (equivalence classes of) real valued pth power integrable functions on X. Let $\underset{\sim}{Z}(F)$ denote the set of all polynomials in elements of F which have integral coefficients (the integral polynomials of the title). Let $B(X)$ be the set of all g in $\underset{\sim}{Z}(F)$ with $\|g\|<1$ and set $J(X) = \{x\in X: g(x) = 0 \text{ for all } g \in B(X)\}$. Then we have

Theorem. If $J(X)$ is a set with μ-measure zero then $\underset{\sim}{Z}[F]$ is dense in $L_p(\mu)$.

In many interesting cases the hypotheses of the theorem are satisfied.

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For the second main result of the paper we specialize to the following situation. Let X be a product of subsets (compact) of $\underset{\sim}{R}$ and take F to be the family of coordinate projections. Thus $X = \Pi_{i=1}^{n} X_i$, each $X_i \subset \underset{\sim}{R}$, and $F = \{\pi_1,\ldots,\pi_n\}$ whence π_i, $1 \le i \le n$, is defined by $\pi_i(x_1,\ldots,x_n) = x_i$. Then

Theorem

$$J(\Pi_{i=1}^{n} X_i) = \Pi_{i=1}^{n} J(X_i)$$

where the $J(X_i)$ are defined by taking F to consist of the identity function x alone.

The result also extends to infinite products $X = \Pi_{\lambda \in \Lambda} X_\lambda$.

For more details please see the reprints which I sent to AFOSR.

The second paper is "Approximation by polynomials with integral coefficients in several complex variables" in "Approximation Theory III," E. W. Cheney (ed.), pp. 389-404. Here, by a polynomial with integral coefficients we mean a function of the form

$$\sum_{k_1,\ldots,k_n = 0}^{m} A_{k_1,\ldots,k_n} z_1^{k_1} \ldots z_n^{k_n}$$

where the a's belong to a fixed but arbitrary discrete subring A of the complex numbers $\underset{\sim}{C}$ of rank 2 (the Gaussian integers for example). Let X be a compact subset of the space $\underset{\sim}{C}^n$ of n complex variables. We say that X is

Mergelyan if any complex valued function which is continuous on X and holomorphic on X^0 (interior of X) is in $\underset{\sim}{C}[Z]^-$, the uniform closure of the polynomials in the coordinate functions $Z_1,\ldots,Z_n$. For example (Mergelyan) a compact subset X of $\underset{\sim}{C}$ is Mergelyan if and only if $\underset{\sim}{C}\setminus X$ is connected. The main result of the paper is

Theorem. If X is a Mergelyan subset of the open unit polydisk ($D = \Pi_{i=1}^{n} [|Z|<1]$) and $0\in X^0$ then a complex valued function f on X is uniformly approximable by polynomials with integral coefficients if and only if f is continuous on X, holomorphic on X^0 and the coefficients of its power series expansion about 0 lie in A.

The condition $X \subset D$ can be replaced by $X \subset a + \underset{\sim}{D}$ where a is any element of A.

Some additional necessary conditions for approximability are given as well as some theorems to show that Mergelyan sets exist. The importance of the notion of polynomial convexity is also shown.

For more details reprints have been recently received and are available.

The book that was completed is "Approximation by Polynomials with Integral Coefficients", Mathematical Surveys, No. 17, American Mathematical Society, Providence, RI 1980. This contains a complete exposition of results in the field of

approximation by polynomials with integral coefficients. As such it will be very valuable to any researcher who needs access to the known results in the field. It was started on an earlier grant (AFOSR 71-2030). A copy now exists in all the major research libraries of the world.

One last publication written during the grant period is "Approximation by polynomials with integral coefficients and digital filter design", in Numerical Methods of Approximation Theory, International Series of Numerical Mathematics 52, Birkhäuser Verlag, Basel, 1980, pp. 111-116. This is a note presented at an Oberwolfach meeting. In it we indicate how approximation by polynomials with integral coefficients enters into digital filter design. The hope is to stir up interest in this application among researchers in approximation theory.

The major goal of the grant was to see if an algorithm for finding a polynomial of best approximation with integral coefficients could be developed. This is what is actually needed in digital filter design. The research produced is a step in that direction but the goal itself was not attained.

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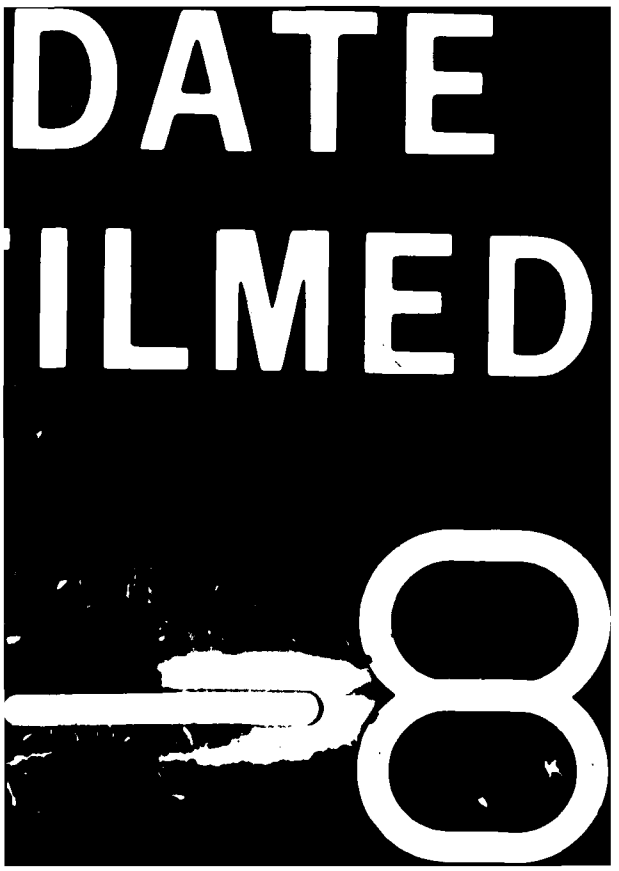
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